AN ANALYSIS OF THE DECISION PROCESS

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ABSTRACT

The purpose of this paper is to formulate a simple analysis of the decision process, in order to optimize a decision over a time interval. In particular, a military decision process is analyzed in relation to input and output parameters. These parameters vary with time according to the values held by those making a decision. Values also change in both short intervals and long intervals and an analysis of the value trends, by experiment, is made. From such trends, projection into the future can be made, such that optimization of the decision process can be established. An optimization rule for the decision process is presented, utilizing trend analysis from experimental data of the values of input and output parameters.

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1. Introduction.

In general, the decision process is the optimization of a resultant which the decider has determined is necessary to obtain. A decision may be made considering both short and long periods of time. Experience and judgement association in relation to the resultant is often relied upon heavily when projecting a decision into the future. The results of a decision may or may not be satisfactory depending upon the ability of the decision maker.

As the process of deciding is dependent upon inputs and outputs to the problem, how one evaluates the values associated with the parameters, will affect this optimization. As values change with time, the trends of change over both short and long time periods is essential prior to the projection of the decision process into the future.

This thesis is a modest evaluation of the projection of values, their relative importance, and optimization of a decision rule over a future time interval. A military application to the decision rule optimization is analyzed at length.

2. Varying Values of Man.

A problem facing men of all times in the decision process is the varying values upon which the action is to be taken or determined. When a restriction or rule is placed upon the decision process, the decision becomes a matter of solving a problem and not one of making a free choice. Such problem solving becomes a matter of acceptance of a set of values, not necessarily belonging to those making the decision. On the contrary, free choice behavior becomes a matter of value selection.

Values become a function of time with a continuously changing slope or derivative which varies with the attitudes of the individual at a particular time. When a particular choice need be made, the individual, with or without conscious awareness, lists and evaluates value parameters of the problem and acts to optimize a particular value. An evaluation of the relative importance of the individual values of these parameters is made as well, and the optimization becomes a linear separation process (i.e., determining the differences for a particular time).

Therefore, the resultant becomes the difference between the factors which cause a gain of the resultant and those which cause a loss of the resultant. Associated with each loss or gain is a relative importance of the existence of the value of the loss or gain. In statistical situations, the relative importance becomes an actual probability determinable by classical means. But, in emotional evaluations for establishing utility values, the relative importance is an existence measure and not necessarily a probability.

In the latter case, probability distributions are very difficult to establish, and for this reason, this thesis is primarily concerned with relative importance. In the simple example of profit determination, the gains can be thought of as credits and the losses as debits. Each credit and debit has a relative importance that are equal to each other and is generally considered to be unity. However, in non-material problems, the relative importance is not likely to be unity. The loss and gain factors which can be considered as outputs and inputs respectively, have values. The value scale may be highly individualistic in nature, but transformation to a mathematical scale is possible. Determination of the resultant follows the relationship:

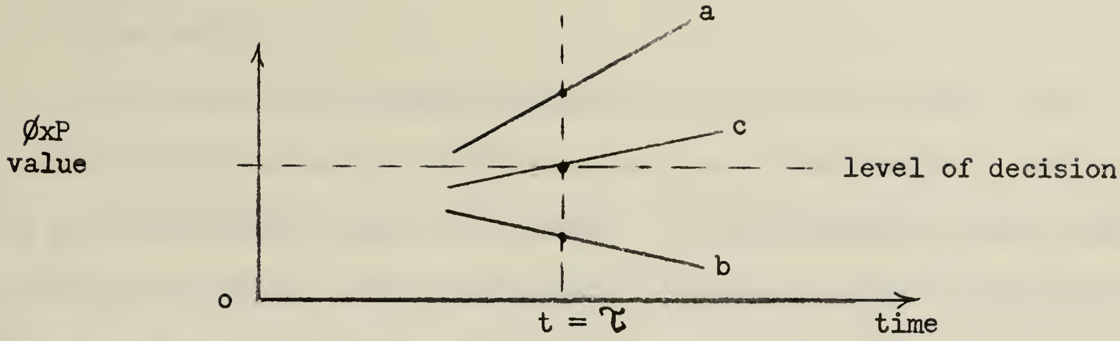
$$\phi(\text{result})P(\text{result})=\phi(\text{inputs})P(\text{inputs})-\phi(\text{outputs})P(\text{outputs}), \quad (1)$$

where ϕ represents the value of the input or output parameters, and P represents its relative importance.

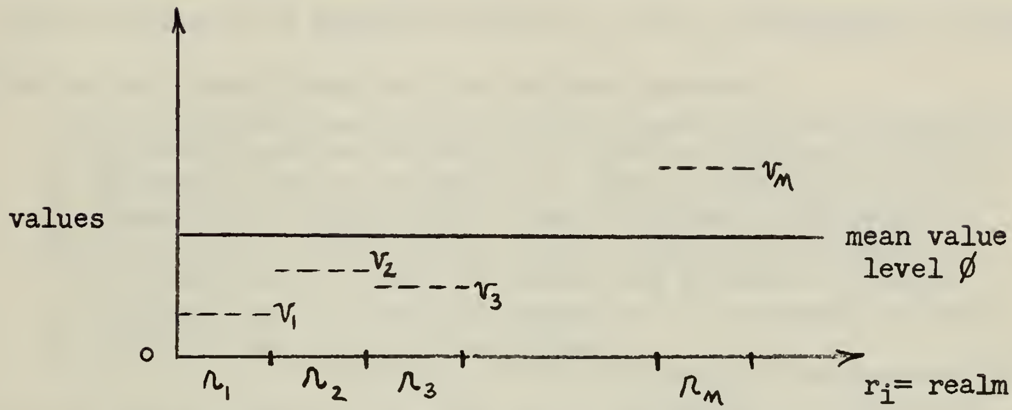
If as in figure 1, slope "a" represents the input term of equation 1 as a function of time, and "b" the output term, the difference gives a result "c" with a particular slope. Thus a resultant for time τ is attained. The words result and resultant will be considered to be identical in definition to the terms decision level and level of decision, and these phrases will be used interchangeably. In decision problems, where optimization of the result is desired, one must consider over a time interval all such "c" results for all τ. This approach will provide an optimal value for the result within the time period in question.

As the values of the inputs and outputs to the problem become one of individual evaluation, the decision level will vary with individuals. Therefore, the decision level becomes a function of levels of values of the individual and also a function of the individual interpretation of of what constitutes inputs and outputs to the problem.

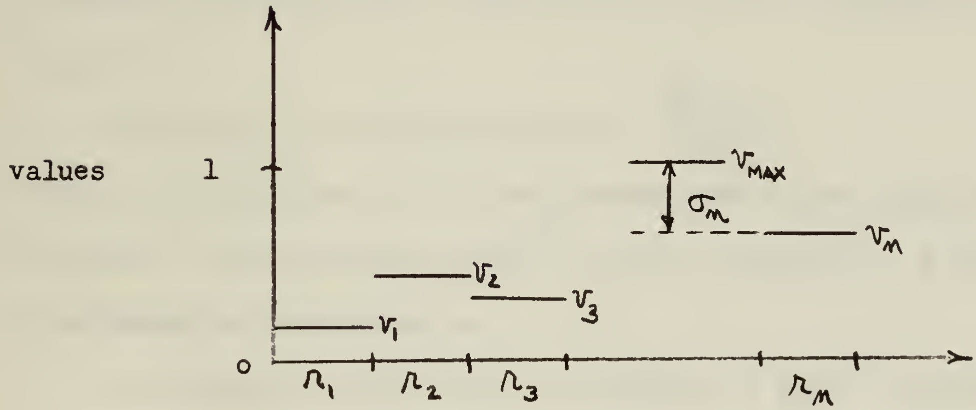
In mechanical problem solving type situations, the input and output values are often clear statistics and thus the level of decision becomes a common fact without regard for whom the decision maker might be. But when values of items like human life, prestige, honor, loyalty are factors the level of decision varies with each decision maker, because of individual level of values. Thus, for one to be confident in a decision level attainment, one must be confident in his value level.



LEVEL OF DECISION
FIGURE 1



MEAN VALUE LEVEL FOR AN INPUT OR OUTPUT AT t= τ
FIGURE 2



DIFFERENCES IN REALM VALUES AT t= τ
FIGURE 3

3. Value Levels.

How should an individual determine his levels of values? This question has been asked by mankind as long as it has existed. But, this by no means implies there is no answer. As man progresses toward understanding of values, no set rule appears evident, as this too would become a matter of problem solving. Choice behavior of values is individualistic in nature, there being no exact common formulation. However, a person can study and evaluate his own standards as a function of time.

The following is a suggested method of how an individual or executive may look at his value level for the decision process.

> For each input or output of the process such as religion, philosophy, music, mathematics there exists areas of influence or various aspects (eg: calculus) applicable to these areas. Each aspect (later referred to as realm) for each input and output factor has a value to the individual (may be zero) in relation to the overall average value of the input or the output.

Now as this value changes with time, it is essential to consider an instantaneous mean value for a particular time $t = \tau$.

Figure 2 shows a possible graph for a particular time as to how an individual may ascertain a mean value level for a particular input or output.

Definition 1: Mean Value Level = $\dfrac{\sum_{i=1}^{m} v_i}{m}$.

where v_i is the value of the realm determined by the individual on his own scale. For simplicity the i^{th} realm is denoted r_i, a symbol having no mathematical significance.

A particular problem in this situation is that of determining the v_i and which r_i to use. To answer this becomes a matter of subjecting individuals to selection rules and general problem solving. Thus, each decision maker must determine his scale and realm selection. Only criticism of the final decision will bear out his values. A suggested manner of determining mean value levels will be outlined in sections 4 and 5.

The mean value level for all t can in theory be plotted, and a value curve for each input and output as a function of time can be established. The individual may project himself into the future by past evaluation of the short term and long term analysis of the mean value levels and their relative importance.

4. Individual Level Assessment.

As previously mentioned, exact rules are difficult to pattern on all individuals to obtain experimentally a mean value level for each input or output to the problem. But, in theory, one can set up a criterion which can, for an individual, be tested under laboratory conditions. Let a problem for a decision be established such that the decision maker is able to determine the factors or inputs and outputs, for a desired result to be attained. Considering individually each input and output, the decision maker must ascertain the realms involved for each of these factors. As will be presented later, these inputs and outputs and realms may be either determined in advance by an outsider, or by the decision maker.

If the individual evaluates on his own scale, the most valuable realm then by reassigning a value of 1 (or 10), all that remains is to find v_i (or the value difference σ_i as in figure 3) for all other realms.[1] Once this is done the mean value level can be established.

This mean value level is related only to the specific input or output. A relationship between the respective inputs and similarily respective outputs is needed. If the individual evaluates on his own scale the most valuable input and output, then by reassigning a value of 1 (or 10), all that remains to be found is the value differences between inputs and between outputs. In the process of rescaling, the mean value level of each input becomes a percentage value of the most valuable in-

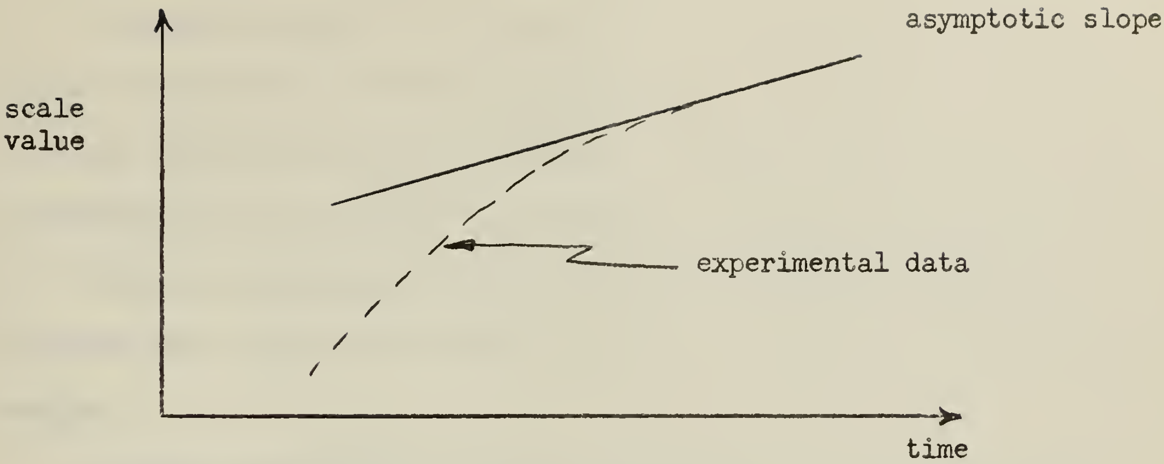
[1]Perhaps a method of obtaining σ_i is by a payoff matrix with use of small amounts of money such that for certain amounts, the individual becomes indifferent between v_i and v_n thus setting a scale for σ_i. This method or others will depend upon the individual but the level can be established. A simple method is presented in the next section.

put and the establishment of a reference scale is complete. This is similarily true for the outputs. As the relative difference between the inputs and outputs is all that is essential in the optimization process, scaling between the two is unnecessary. A means of determining a relative importance related to these adjusted mean value levels will be presented later.

The importance of level selection of values becomes essential to the decision maker. Each decider must find his value levels, so that the level of decision has confidence. The mean value levels when coordinated to a common scaling (becoming adjusted mean value levels) will provide the necessary numerical values for inputs and outputs for the determination of the level of decision. From the experimental data, mean value levels and their relative importance will sometimes asymptotically approach a slope which is a function of time (see figure 4). The more consistent and stable the decision maker is in his projection, the more stable this function becomes.

A basic laboratory experiment follows in section 5, as a suggested simple procedure for determining trends of both mean value levels and their relative importance. Two experiments are to be considered: the first, has all inputs, outputs and realms determined by the testee; the second, has predetermined fixed inputs, outputs and realms for individual evaluation. The time intervals for both experiments are different. Results of the experiment are outlined in section 6.

One or the other of these experimental evaluations holds for all decision processes, and in the following sections outlining the optimization of the decision level, it is assumed that for the practical short-time (several days) decisions, the decider has established his level of



DETERMINING SLOPE (TREND)
FIGURE 4

values. Only a practical optimization rule will be presented.

5. Laboratory Experiment for Value Level Determination.

The general purpose of the experiments is to ascertain the mean value levels and the relative importance of the parameters as functions of time. These functions are essential for projection into the future. Depending upon the stability of projection of thought of the testee into the future, an asymptotic slope as a function of time can sometimes be obtained (see figure 4) for input and output parameters in the decision region.

In these experiments, the testee will use a set scale of 0 to 10 for evaluation of the realms, inputs and outputs. Although the testee could use his own scaling factors, this scale is utilized to overcome the necessity of transforming value assignment to a standard scale and for simplicity of analysis.

Definition 1: $\emptyset$ = mean value level = $\dfrac{\sum_{i=1}^{m} v_i}{m}$.

This definition was presented in section 3. v_i is the value assigned each realm. A subscript t, where t = 1, 2, 3, ---, n will be utilized on $\emptyset$ to denote test number in the sequence of the experimentation.

Definition 2: $\emptyset'$ = adjusted mean value level =

(mean value level)(value assignment to input or output) =

$\emptyset$ (value assignment to input or output).

A subscript t as discussed above will also be utilized on $\emptyset'$. Definition 2 establishes a relationship between all inputs and similarily between all outputs.

If the testee is asked at each testing to subjectively estimate the percentage of concentration (equivalent to relative importance) he utilizes on each realm for the inputs and outputs, a weighted mean value level is established.

Definition 3:

Weighted mean value level = $\sum$(realm weight)(%realm concentration).

From definition 3 it follows that:

$$\text{weighted mean value level} = C\phi' \tag{1}$$

where C represents a weight factor.

Definition 4: C = relative importance of ϕ' and from section 2 that:

$$C = P \equiv P\ (\text{input})^{1}. \tag{2}$$

For experimental purposes, the administered tests are of such a nature that relative importance of the factors must be considered in its broadest sense, and not as an actual statistical probability or proability function. The validity of this definition is reasonable, as the relative importance becomes significant when stability is sought over a time interval. In relatively short intervals of time, the relative importance may fluctuate over a large interval, depending upon the emotional stability of the individual and the environmental factors influencing these emotions. Additionally, a person's fluctuations may be due to indecision, but once he has analyzed the factors and established the soundness of his choice, a trend may be evident. Therefore, from 1 and 2:

$$P \equiv P(\text{input}) = \sum \frac{(\text{realm value})(\%\text{realm concentration})}{\phi'}, \tag{3}$$

$$P\phi' = \sum(\text{realm value})(\%\text{realm concentration}). \tag{4}$$

A subscript t as previously defined will be utilized on P and $P\phi'$ as well.

If in the test, the testee makes a subjective estimate of the time

[1] A similar statement holds for the output parameters.

the result is attained, this estimate also becomes a measure of relative importance. Therefore, from a trend analysis and by use of equation 1, section 2, the value of the result (ϕ(result)) can be determined. The product of the values of the input and output values and their relative importance for substitution in this equation are the relationships established by 4 above.

5.1 Experiment 1

Part I

Objective: To determine the mean value levels of the input and output parameters in a decision process optimization of a desired resultant factor. As the mean value level is a function of time, for each parameter, an asymptotical functional relationship is desired. The test must be conducted at discrete time intervals.

Procedure:

1. Explain to testee that the result of any decision process is related to the values of the inputs and outputs. The inputs contribute to maximizing and the outputs contribute to minimizing the resultant.

2. State the desired result to be optimized.

   In the present experiment it is: happiness

3. Have testee list inputs and outputs.

4. Have testee list inputs in descending order of preference. Repeat for outputs.

5. Have testee numerically assign an ingredient weight from 0 to 10 (integer or non-integer) on each input and output.

6. For each input have testee list in descending order of preference the realms of the inputs. Repeat for outputs.

7. Have testee assign numerically an ingredient weight from 0 to 10 for each realm.

8. Repeat at discrete time intervals.

Method:

1. For each test, determine the mean value level for each input and output as in definition 1.

2. Establish the adjusted mean value level for each input and output as in definition 2.

Part II

Objective: To determine the relative importance of each of the adjusted mean value levels of the inputs and the outputs.

Procedure:

1. For each realm ask testee to estimate the percentage of each input to which his effort is concentrated in thought. Repeat for each output.

2. Have testee state what percentage of the time he is happy.[1]

Method:

1. Determine P, the relative importance from equation 3 for each input and output.

2. As the relative importance of P (result) is subjectively asked for, all factors of equation 1, Section 2 are known except $\emptyset$ (result). Substitute and solve for $\emptyset$ (result).

Results:

The mean value level, adjusted mean value level and the relative importance of the input and output factors are now known. Retesting

[1]Relative importance estimate of the percentage of time the result is attained.

over time intervals will provide trends as functions of time. From these trends optimization of the decision level is now possible. See Appendix I for the actual test sheet administered in this experiment.

5.2 Experiment 2.

Objective: Same as experiment 1.

Procedure: Give testee a sheet with specified realms, inputs and outputs which are the factors of success at work. Carry out steps 5 through 8, part I; step 1, part II of experiment 1. Ask for a subjective percentage that success at work is attained. (relative importance)

Method: Same as experiment 1.

Results: The results should be similar to those of experiment 1, but due to the specific nature of the test, experiment 2 will eliminate the possibility of a "random walk" result that may be present in experiment 1. See Appendix II for test sheet used.

5.3 Conclusions: The subjective evaluation in both experiments may only give subjective results depending upon the stability of the testee as a decision maker. If an asymptotic trend is attained, then objective results are evident and projection into the future reliable.

6. Results of Experiment.

6.1 Experiment 1

The experiment was conducted with two testees, a housewife and a U.S. Navy enlisted man over discrete intervals of 1, 2 and 4 days with resulting mean value levels, adjusted mean value levels and their relative importance as shown in tables 1 and 2. All inputs and outputs for both, remained toward the end of the period, none being added or deleted, but, their assigned values changed with time. The realms existed consistently for the enlisted man throughout the test, but this was not so for the housewife. In general a random walk type of result for the inputs and outputs existed for both with no asymptotic approach as a function of time. See figure 5 for a plot of input A for the enlisted man.[1]

The lack of an establishment of a trend can be credited to the lack of projection of values with stability over a time period. From the data, both testees seemed to demonstrate a tendency to be concerned for short time intervals, indicating this instability of values. The test may be criticized for its broad spectrum which could cause discrepancies of input and output evaluation. However, when the enlisted man had conducted the test for the fourth, fifth and sixth times, all inputs, outputs and realms remained, without addition or deletion. Still a random result occurred except for input B which did reach a constant slope of 8.0, 64.0, and 0.125.

[1]Although input A was named specifically by a noun by the testee, and psychological analysis is not the purpose of the thesis, the inputs and outputs will be given letter captions. Trend analysis is the essential purpose.

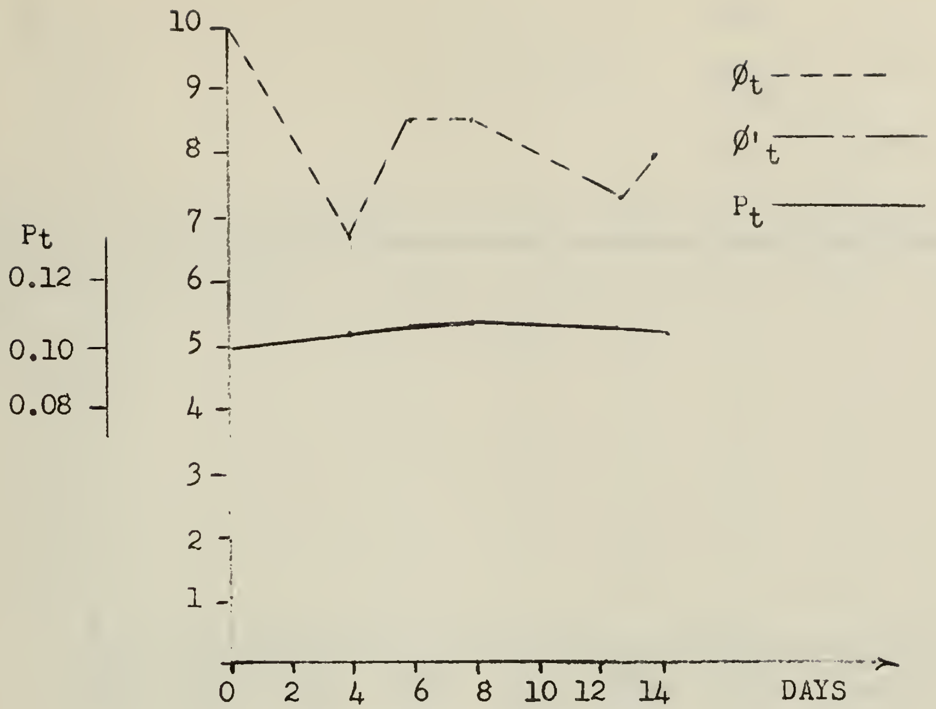
This indicates that input B value became firmly established within the man's mind as a function of time.

6.2 Experiment 2.

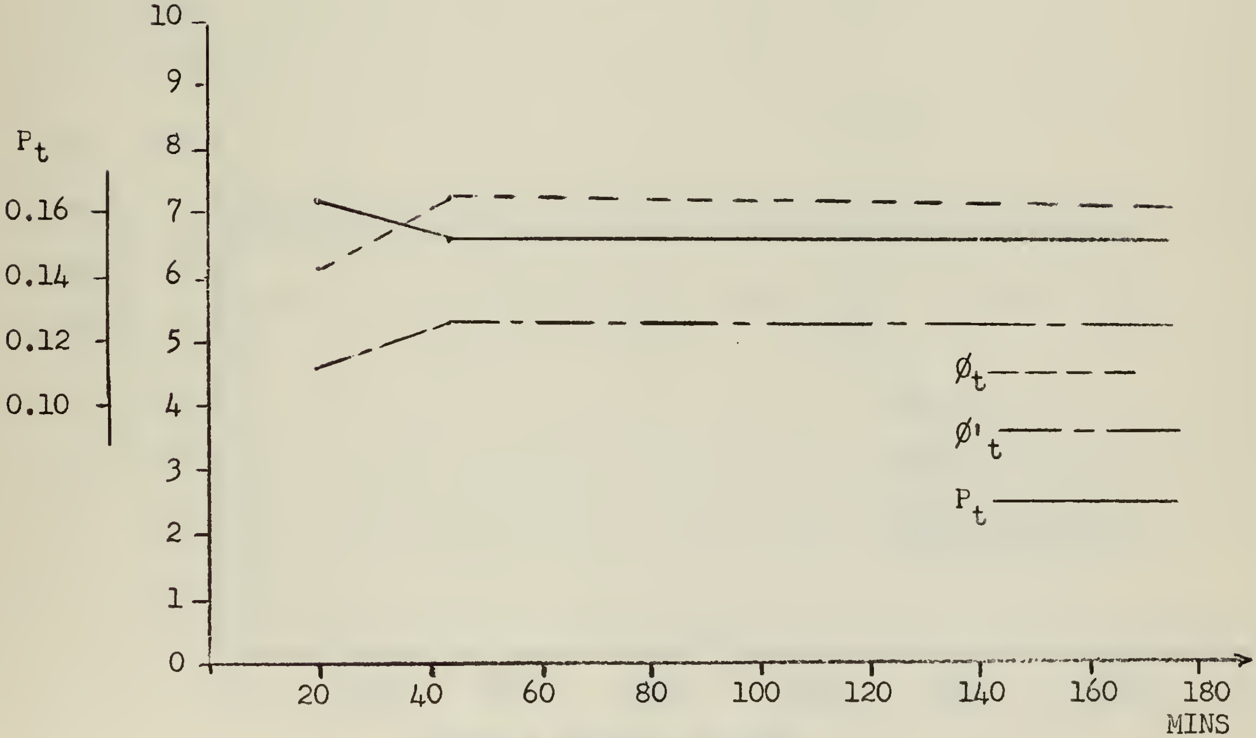
This testee, a more mature woman, greatly familiar with weight evaluation and measure theory was given the test over the time interval as shown in table 3. All inputs and outputs except input E and output Y reached a constant as a function of time as shown in table 3. Figure 6 shows the results for input A over the first short time interval. Figures 7 and 8 show the results for input A over other intervals. This is precisely the asymptotic approach desired for prediction into the future. The stability of values of the testee is indicated.

6.3 Comments.

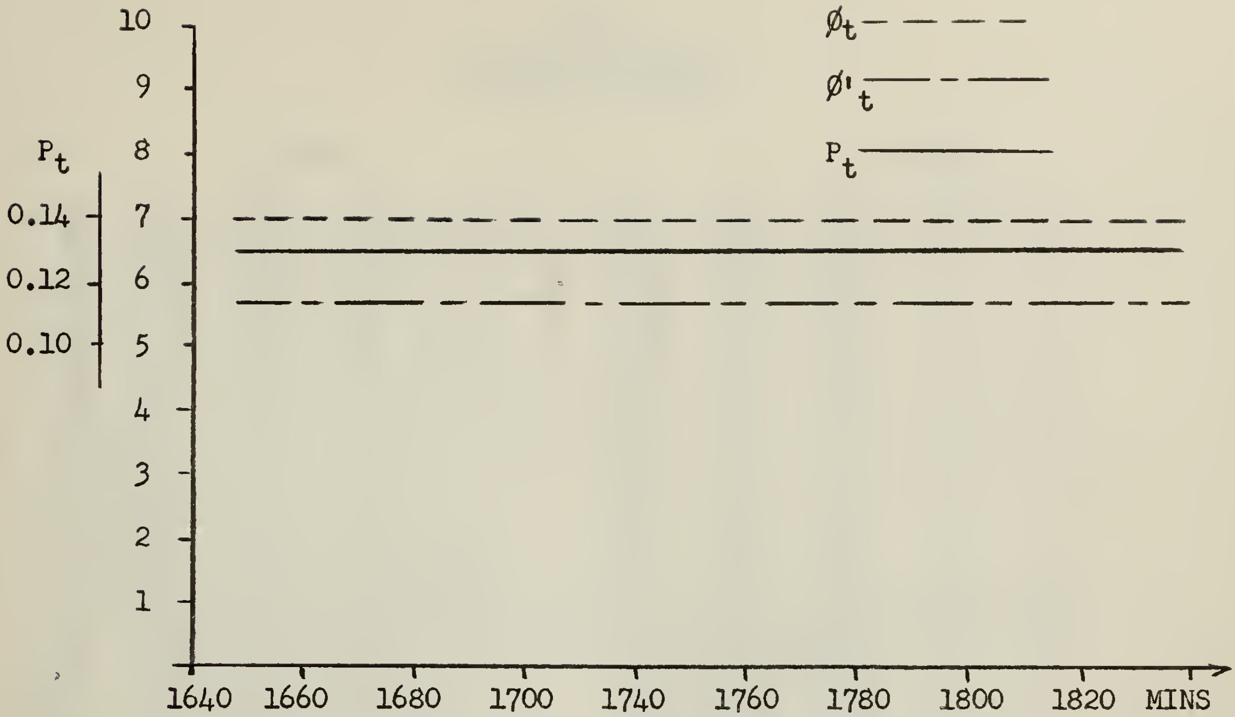
By no means is it implied that the experiment will always produce mean value levels and their relative importance which will approach asymptotically a slope as a function of time. This result is highly individualistic, but what is essential is that under the conditions of stability of the testee, the test produces as designed. Sample worksheet and calculations are found in Appendices 3 and 4.



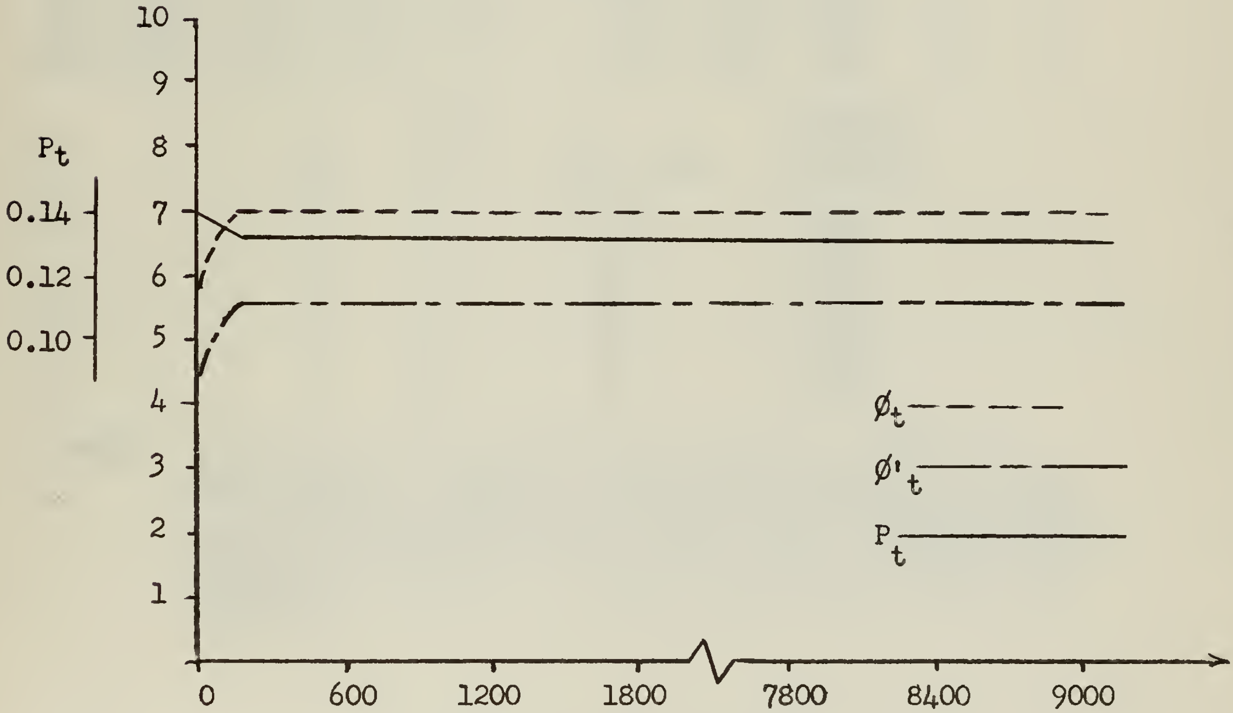
INPUT A RESULTS VS TIME
EXPERIMENT 1
ENLISTED MAN
FIGURE 5



INPUT A RESULTS VS TIME
$0 \leq t \leq 200$
EXPERIMENT 2
FIGURE 6



INPUT A RESULTS VS TIME
$1640 \leq t \leq 1800$
EXPERIMENT 2
FIGURE 7



INPUT A RESULTS VS TIME
FOR TOTAL TEST PERIOD
EXPERIMENT 2
FIGURE 8

TABLE 1
SUMMARY EXPERIMENT 1
U.S.N. ENLISTED

	INPUTS							OUTPUTS		
	A	B	C	D	E	U	[illegible]	W	X	Y
ϕ_1	10.0	9.00	10.0	9.00	6.33	9.50	9.50	7.67	10.0	8.00
ϕ_2	6.67	8.00	8.00	7.67	0	7.33	7.50	8.50	0	0
ϕ_3	8.33	9.50	10.0	0	0	7.00	9.00	7.00	0	0
ϕ_4	8.33	8.00	7.50	6.33	0	8.66	9.50	8.50	0	0
ϕ_5	7.37	8.00	8.00	7.67	0	6.67	9.00	7.00	0	0
ϕ_6	7.67	7.00	8.00	6.67	0	6.67	8.00	7.00	0	0
ϕ'_1	100	72.0	80.0	54.0	31.7	95.0	85.5	50.0	53.7	24.0
ϕ'_2	66.7	64.0	64.0	38.4	0	73.3	37.5	42.5	0	0
ϕ'_3	83.3	76.0	90.0	0	0	70.0	81.0	42.0	0	0
ϕ'_4	83.3	64.0	60.0	38.0	0	86.6	85.5	59.5	0	0
ϕ'_5	73.7	64.0	64.0	46.0	0	66.7	81.0	49.0	0	0
ϕ'_6	76.7	56.0	64.0	39.0	0	66.7	72.0	42.0	0	0
P_1	0.100	0.128	0.125	0.167	0.276	0.102	0.111	0.200	0.149	0.333
P_2	0.103	0.125	0.128	0.214	0	0.104	0.200	0.205	0	0
P_3	0.106	0.127	0.111	0	0	0.104	0.112	0.173	0	0
P_4	0.106	0.128	0.125	0.147	0	0.105	0.112	0.151	0	0
P_5	0.103	0.128	0.125	0.159	0	0.118	0.113	0.147	0	0
P_6	0.104	0.129	0.125	0.159	0	0.111	0.114	0.172	0	0
$P\phi'_1$	10.0	9.20	10.0	9.00	8.75	9.70	9.50	8.00	10.0	8.00
$P\phi'_2$	7.00	8.00	8.20	8.20	0	7.60	7.50	8.55	0	0
$P\phi'_3$	8.80	9.60	10.0	0	0	7.25	9.10	7.20	0	0
$P\phi'_4$	8.80	8.20	7.50	5.60	0	9.05	9.60	8.95	0	0
$P\phi'_5$	7.60	8.20	8.00	7.30	0	7.90	9.20	7.20	0	0
$P\phi'_6$	8.00	7.20	8.00	6.20	0	7.40	8.20	7.20	0	0

TEST	TIME DAYS	%=P(result)	RELATIVE ϕ(result)
1	1	60	2.92
2	4	60	12.90
3	6	60	8.07
4	8	60	4.17
5	11	60	11.30
6	12	60	11.00

NOTE: 0 indicates lack of existance of input or output at each testing. Inputs A through E and outputs U through Y, listed in letter form. Actual input and output nouns deliberately omitted to keep test matter confidential as promised to testee. For similar inputs and outputs see Appendix 3.

TABLE 2
SUMMARY EXPERIMENT 1
HOUSEWIFE

	INPUTS						OUTPUTS					
	A	B	C	D	E	F	U	V	W	X	Y	Z
$\emptyset_1$	9.00	8.50	9.50	7.50	8.50	0	8.00	7.50	8.00	9.00	0	0
$\emptyset_2$	9.33	7.50	8.50	9.00	7.50	8.67	9.00	0	9.00	9.00	8.00	8.50
$\emptyset_3$	8.67	8.33	9.50	9.00	9.00	9.20	9.50	9.00	8.67	8.67	8.40	9.00
$\emptyset_4$	9.00	8.67	9.00	9.00	9.33	9.50	9.00	8.50	9.50	9.00	9.50	8.50
$\emptyset_5$	8.75	8.67	8.50	9.00	8.33	8.00	8.50	8.50	8.33	8.75	9.00	9.00
P'_1	90.0	76.5	38.0	45.0	59.5	0	56.0	45.0	64.0	80.0	0	0
P'_2	93.3	60.0	59.5	45.0	60.0	78.0	72.0	0	72.0	90.0	72.0	51.0
P'_3	86.7	75.0	66.5	45.0	72.0	73.8	57.0	36.0	78.0	86.7	75.6	72.0
P'_4	90.0	79.0	72.0	54.0	65.3	85.5	72.0	59.5	85.5	90.0	76.0	51.0
P'_5	87.5	77.0	68.0	54.0	58.3	72.0	68.0	59.5	75.0	87.5	81.0	54.0
P_1	0.100	0.110	0.250	0.180	0.143	0	0.154	0.170	0.137	0.116	0	0
P_2	0.105	0.125	0.145	0.206	0.125	0.115	0.128	0	0.127	0.102	0.114	0.169
P_3	0.104	0.117	0.145	0.202	0.126	0.116	0.167	0.254	0.114	0.099	0.116	0.128
P_4	0.093	0.114	0.129	0.170	0.143	0.112	0.131	0.143	0.112	0.101	0.126	0.178
P_5	0.108	0.111	0.129	0.170	0.152	0.112	0.134	0.145	0.117	0.102	0.113	0.168
$P\emptyset'_1$	8.99	8.65	9.65	8.10	8.50	0	8.60	7.65	8.75	9.30	0	0
$P\emptyset'_2$	9.40	7.50	8.60	9.30	7.50	8.95	9.20	0	9.10	9.20	8.20	8.60
$P\emptyset'_3$	8.95	8.75	9.65	9.10	9.10	8.60	9.65	9.15	8.75	8.55	8.75	9.25
$P\emptyset'_4$	8.35	9.00	9.30	9.20	9.35	9.60	9.40	8.50	9.60	9.10	9.60	9.10
$P\emptyset'_5$	9.50	8.55	8.80	9.20	8.95	8.10	9.10	8.60	8.75	8.95	9.20	9.10

TEST	TIME DAYS	%=P(result)	RELATIVE Ø(result)
1	1	70	13.70
2	4	65	10.70
3	6	65	0.08
4	11	65	-0.83
5	12	65	-1.00

NOTE: 0 indicates lack of existance of input or output at each testing.

TABLE 3
SUMMARY EXPERIMENT 2

	INPUTS								OUTPUTS	
	A	B	C	D	E	U	V	W	X	Y
$\emptyset_1$	5.83	6.67	5.50	3.75	2.25	3.20	6.33	6.67	3.00	3.25
$\emptyset_2$	7.00	6.17	4.90	3.87	3.25	3.50	6.17	7.00	4.25	3.50
$\emptyset_3$	6.50	6.17	5.20	4.50	3.00	3.60	6.00	7.00	4.50	3.25
$\emptyset_4$	6.67	5.33	4.40	3.25	3.00	3.20	6.00	6.67	4.25	3.25
$\emptyset_5$	7.00	6.00	4.20	4.00	2.75	4.20	6.00	7.00	4.25	2.75
$\emptyset_6$	7.00	6.00	4.20	3.25	2.75	4.00	6.00	7.00	4.00	2.75
$\emptyset_7$	7.00	6.00	4.30	3.25	2.75	4.00	6.00	7.00	4.00	2.75
$\emptyset_8$	7.00	6.00	4.30	3.25	2.75	4.00	6.00	7.00	4.00	2.75
$\emptyset_9$	7.00	6.00	4.30	3.25	2.75	4.00	6.00	7.00	4.00	2.75
$\emptyset'_1$	43.8	33.3	27.8	13.1	2.25	16.0	47.5	50.0	15.0	8.10
$\emptyset'_2$	52.5	37.0	29.4	19.4	8.13	14.0	37.0	52.5	21.2	7.00
$\emptyset'_3$	51.9	49.3	36.4	27.0	7.50	18.0	42.0	56.0	27.0	8.12
$\emptyset'_4$	53.3	37.4	26.4	16.3	3.00	12.8	42.0	53.3	25.5	6.50
$\emptyset'_5$	56.0	42.0	25.2	20.0	6.87	25.2	42.0	56.0	21.3	6.87
$\emptyset'_6$	56.0	42.0	25.2	16.3	6.87	24.0	42.0	56.0	20.0	6.87
$\emptyset'_7$	56.0	42.0	25.8	19.5	13.75	24.0	42.0	56.0	24.0	13.8
$\emptyset'_8$	56.0	42.0	25.8	16.3	6.87	24.0	42.0	56.0	20.0	6.87
$\emptyset'_9$	56.0	42.0	25.8	16.3	11.0	24.0	42.0	56.0	20.0	11.0
P_1	0.142	0.200	0.216	0.317	1.00	0.234	0.139	0.140	0.254	0.481
P_2	0.134	0.177	0.179	0.216	0.468	0.275	0.172	0.136	0.226	0.556
P_3	0.135	0.128	0.155	0.174	0.507	0.255	0.150	0.130	0.174	0.468
P_4	0.131	0.155	0.163	0.230	1.00	0.293	0.150	0.132	0.192	0.600
P_5	0.130	0.150	0.185	0.225	0.480	0.183	0.150	0.130	0.225	0.480
P_6	0.130	0.150	0.184	0.240	0.480	0.196	0.147	0.128	0.250	0.480
P_7	0.130	0.150	0.169	0.200	0.240	0.196	0.147	0.128	0.208	0.240
P_8	0.130	0.150	0.169	0.240	0.480	0.196	0.147	0.128	0.250	0.480
P_9	0.130	0.150	0.169	0.240	0.272	0.196	0.147	0.128	0.250	0.272
$P\emptyset'_1$	6.25	6.67	6.00	4.15	2.55	3.75	6.60	7.00	3.80	3.90
$P\emptyset'_2$	7.05	6.55	5.25	4.18	3.80	3.85	6.35	7.15	4.80	3.90
$P\emptyset'_3$	7.03	6.35	5.65	4.70	3.80	4.05	6.30	7.25	4.70	3.80
$P\emptyset'_4$	7.00	5.80	4.30	3.75	3.00	3.75	6.30	7.05	4.90	3.90
$P\emptyset'_5$	7.30	6.30	4.65	4.50	3.30	4.65	6.30	7.30	4.80	3.30
$P\emptyset'_6$	7.30	6.30	4.65	3.90	3.30	4.70	6.15	7.15	5.00	3.30
$P\emptyset'_7$	7.30	6.30	4.70	3.90	3.30	4.70	6.15	7.15	5.00	3.30
$P\emptyset'_8$	7.30	6.30	4.70	3.90	3.30	4.70	6.15	7.15	5.00	3.30
$P\emptyset'_9$	7.30	6.30	4.70	3.90	3.30	4.55	6.15	7.15	5.00	3.30

TEST	TIME MINS	%=P(result)	relative Ø(result)
1	19	50	1.14
2	46	60	1.30
3	96	65	2.20
4	173	70	-2.93
5	1647	80	-0.38
6	1674	80	-1.06
7	1717	80	-1.00
8	1781	80	-1.00
9	8981	80	-0.81

7. Development of Practical Theory for Optimizing Level of Decision

Previously we have been concerned with the determination of the mean value levels of an individual. Each decision maker can conduct a test as previously outlined, prior to optimizing the decision level. It is required that the individual either subjectively evaluate these factors, use a test providing trends or mathematically determine them from clearly defined available statistics. As the former types are more generally necessary, a trend analysis as previously discussed will likely be required. It is assumed at this point, that mean value levels and their relative importance have been ascertained.

Referring to figure 1, for a particular course of action, optimization of the resultant can be determined over a time interval provided the input and output factors and their relative importance are known functions of time. This section outlines the mathematics for this optimization process.

Assumptions:

1) At $t=t_o$ input and output factors are clearly defined and are not deleted over the time interval $\left[t_o, t_n\right]$. No additional factors are introduced in the interval at a later time.
2) The decision for action must occur within the interval $\left[t_o, t_n\right]$.

7.1 Input and Output Factors

These factors represent gains and losses upon which the optimization of the level of decision is based. These factors are functions of time as shown in the trend analysis.

Let $\beta_1^o, \beta_2^o, ----, \beta_m^o$ represent the relative importance of the mean value levels of these inputs and outputs at $t=t_o$. At $t = t_k$ where $o \leq k \leq n$,

$\beta_1^k, \beta_2^k, ---, \beta_m^k$ similarily represents these parameters.

Clearly, from trend analysis:

$$\begin{aligned}
\beta_1 &= \beta_1^o - \beta_1^k = f_1(t) \\
\beta_2 &= \beta_2^o - \beta_2^k = f_2(t) \\
\beta_3 &= \beta_3^o - \beta_3^k = f_3(t) \\
&\vdots \\
\beta_m &= \beta_m^o - \beta_m^k = f_m(t),
\end{aligned}$$

where $f_1(t)$, $f_2(t)$, $f_3(t)$, ---, $f_n(t)$ are monotone increasing, monotone decreasing or constant over the interval.

7.2 Mean Value Levels of the Parameters

Each input or output factor has an associated mean value level, either determinable or non-determinable from a trend analysis. It is assumed in this theory that optimization can be accomplished with only one non-determinable factor.

Let ϕ_1, ϕ_2, ϕ_3, --- ϕ_n, all functions of time be the mean value levels associated with $\beta_1, \beta_2, \beta_3, ---, \beta_m$. Without loss of generality let $\beta_1, \beta_2, ---, \beta_k$ be input factors, $\beta_{k+1}, ----, \beta_{m-1}$ be output factors and β_m be result factor. Recall:

$$\phi(\text{result})P(\text{result}) = \phi(\text{inputs})P(\text{inputs}) - \phi(\text{outputs})P(\text{outputs}) \quad (1)$$

The result factor may also be an input or output factor, and is to be the quantity for desired optimization. Example of results that may be considered are: prestige, human life, profit.

$$\beta_m\phi_m = \beta_1\phi_1 + ----- + \beta_k\phi_k - \beta_{k+1}\phi_{k+1} - ---- - \beta_{m-1}\phi_{m-1}. \quad (2)$$

$\phi_n = g(t)$ as all members on right side of (2) are constants or functions of t. (3)

The decision to act is made when ϕ_n is a maximum or minimum, as the problem dictates.

$$g'(t) = 0. \quad (4)$$

Equation 4 gives a solution $t = \tau$ and $\phi_n = g(\tau)$ is the maximum or minimum value. If the solution fails to fall within the desired interval, an end point solution must be considered.

8. Formulation of a Problem to Optimize Level of Decision

Since the optimization rule has been presented in theory, a specific formulation, followed by numerical analysis is necessary for completeness. A military commander under battle conditions must evaluate all factors of the decision process in relation to mean value levels and their relative importance as a secondary mode for experience. To illustrate the applicability of this thesis within the armed forces, a military problem will be formalized in this section.

Example

General A has k troops at his disposal, l units of equipment costing m dollars. The General has been ordered to take a position within p days expecting n replacement of troops and r replacement of equipment over that period of time. Intelligence information indicates that if the General makes an immediate attack, the enemy forces out weigh his forces sufficiently to cause heavy loss of his forces, but surprise would increase the possibility of success. But in p days less loss of life would occur with less possibility of success. The General wishes to attack such that success is most likely with the restraint of minimizing the loss of life.

It is assumed that the mean value levels and their relative importance are known. Minimizing the loss of life is defined to be that time when the mean value level of human life is a minimum.

TABLE 4

PROBLEM SUMMARY

INPUTS/OUTPUTS	t=o IMPORTANCE	FUNCTION	CHARACTERISTIC	t=p IMPORTANCE
Success	z	γ	decreasing	z'
Loss of Life	y	β	decreasing	y'
Units (equipment)	l	χ	increasing	l'
Troops	k	ψ	increasing	k'
Loss of Units	x	ϵ	decreasing	x'

8.1 Optimizing level of decision.

$\gamma, \beta, \chi, \psi, \epsilon$ are functions of time and represent relative importance of mean value levels for the input and output factors. Similarly let ϕ represent the mean value levels. From equation 1, section 2:

$$\phi(\text{success})\gamma_j = -\phi(\text{life})\beta_j + \phi(\text{troops})\psi_j + \phi(\text{units})\chi_j - \phi(\text{equip loss})\epsilon_j \qquad (1)$$

for $t=t_j$. As previously stated:

$$\phi(\text{success}) = f_1\ (t,S)$$

$$\phi(\text{troops}) = f_2\ (t,S)$$

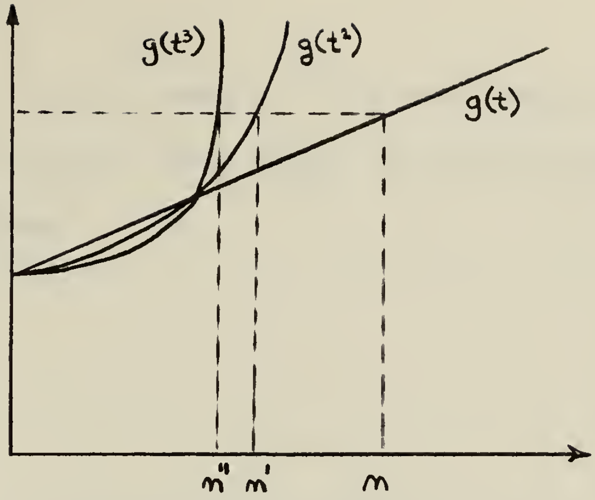
$$\phi(\text{units}) = f_4\ (t,S)$$

$$\phi(\text{equip loss}) = f_5\ (t,S),$$

where S is a variable of the overall strategy and relates the importance of the success of this battle to the overall war effort.[1]

It is assumed since a time delay of p days was imposed upon the accomplishment of this mission, $S= g(t^k)$ where $g(t^k)$ is monatone increasing with k=1. The question remains whether $k=1$ such that $S=g(t)$.

[1] An equivalent factor is the relationship between success at work and total happiness.



S vs time for $S = g(t^k)$ $k = 1,2,3$
Figure 9

Looking at figure 9, if $S=g(t^2)$ then $S \approx g(t)=n'$ is attained when $t=n'$. Similarily $S \approx g(t)=n''$ is attained at $t=n''$ for $S=g(t^3)$. Thus if S were a function of $g(t^k)$ where $k>1$, fewer days n' or n'' would be given. In practice, a military enterprise would not be taken without a margin of assurance. The assumption seems very realistic. Therefore:

$$\begin{aligned}
\phi(\text{success}) &= f_1(t,g(t)) = k_1 f_1(t)\\
\phi(\text{troops}) &= f_3(t,g(t)) = k_3 f_3(t)\\
\phi(\text{units}) &= f_4(t,g(t)) = k_4 f_4(t)\\
\phi(\text{equip loss}) &= f_5(t,g(t)) = k_5 f_5(t).
\end{aligned}$$

From (1):

$$\phi(\text{life}) = k_3 f_3(t)\frac{\psi_j}{\beta_j} + k_4 f_4(t)\frac{\chi_j}{\beta_j} - k_5 f_5(t)\frac{\varphi_j}{\beta_j} - k_1 f_1(t)\frac{\gamma_j}{\beta_j}. \tag{2}$$

$$\phi(\text{life}) = k_3' f_3(t) + k_4' f_4(t) + k_5' f_5(t) + k_1' f_1(t). \tag{3}$$

$$\phi(\text{life}) = f(f_1, f_3, f_4, f_5) = g'(t). \tag{4}$$

$\phi(\text{life})$ becomes a function of time and by minimizing the value of $g'(t)$, $\phi(\text{life})$ takes on its minimum.

$$\frac{d}{dt} g'(t) = 0. \tag{5}$$

Thus, $\phi(\text{life})_{min} = g'(\tau)$ where $t = \tau$ is the solution to (5). If a relative minimum does not occur within the decision period, end point analysis is required.

9. Numerical Problem Analysis

For any theory or formulation to be complete, an example with numerical values is essential. The problem previously formulated will be analized with numerical inputs and outputs. This section is divided into various examples where time is considered continuous over the closed interval $[0,6]$.

9.1 Example 1:

TABLE 5

EXAMPLE 1 SUMMARY

INPUTS/OUTPUTS	t = 0 IMPORTANCE	FUNCTION CHARACTERISTICS	t = 6 IMPORTANCE
Success	$z_o = 0.75$	$\gamma = -t/24 + 3/4$ (decrease)	$z_6 = 0.50$
Life Loss	$y_o = 0.20$	$\beta = -5t/600 + 1/5$ (decrease)	$y_6 = 0.15$
Equipment	$l_o = 0.90$	$\chi = 2t/600 + 9/10$ (increase)	$l_6 = 0.92$
Troops	$k_o = 0.90$	$\psi = 2t/600 + 9/10$ (increase)	$k_6 = 0.92$
Equip Loss	$x_o = 0.40$	$\epsilon = -t/60 + 4/10$ (decrease)	$x_6 = 0.30$

	Case 1:		Case 2:	
$k_1 = 20\%$	ϕ(success)	$= k/6$	ϕ(success)	$= t/6 + 1$
$k_2 = 30\%$	ϕ(troops)	$= k/30$	ϕ(troops)	$= t/30 + 1/5$
$k_3 = 15\%$	ϕ(units)	$= k/20$	ϕ(units)	$= t/20 + 3/10$
	ϕ(equip)	$= k/40$	ϕ(equip)	$= t/40 + 15/100$

It is assumed that from hypothetical trend analysis that the following relative importance functions exist for this example:

$$\gamma = -t/24 + 3/4. \tag{1}$$

$$\beta = -5t/600 + 1/5. \tag{2}$$

$$\chi = 2t/600 + 9/10. \tag{3}$$

$$\psi = 2t/600 + 9/10. \tag{4}$$

$$\epsilon = -t/60 + 4/10. \tag{5}$$

These functions are assumed linear functions of time for simplicity of evaluation. If in fact, another monotome function is known to exist, it may be used without loss of generality. In order that:

$$\phi(\text{success})\gamma = -\phi(\text{life})\beta + \phi(\text{troops})\psi + \phi(\text{units})\chi - \phi(\text{equip loss})\varepsilon$$

can be satisfied, all but ϕ(life) must be established as functions of time. This is the condition of the optimization procedure. From the assumed trend analysis, assume all mean level values (ϕ) except ϕ(life) are ascertainable percentage values of ϕ(success) where:

$$\phi(\text{troops}) = k_1(\phi(\text{success}))$$

$$\phi(\text{units}) = k_2(\phi(\text{success}))$$

$$\phi(\text{equip loss}) = k_3(\phi(\text{success})),$$

where k_1, k_2, k_3 are ingredient weight percentages of success. On first inspection one might deduct that ϕ (life) = $k_4\phi$ (success) where $k_1 + k_2 + k_3 + k_4 = 1$, but this is not necessarily true. There are many other parameters, not considered in the problem which are also elements of success. Furthermore trend analysis will indicate that $k_1 + k_2 + k_3 + k_4 > 1$ might also exist. For this example let:

$$k_1 = 20\%$$

$$k_2 = 30\%$$

$$k_3 = 15\%,$$

where k_1 k_2 k_3 are constant over the interval.

Case 1: Let the assumed trend be such that ϕ(success) = constant k over time interval.

Case 2: Let the assumed trend be such ϕ (success) is linearly increasing from 1 to 2 relative units over a period of 6 days. If immediate success were desired, the attack would be ordered at t = 0. Since a deadline of 6 days is given and ϕ (success) is linear and increasing reaching a maximum of 2 units at the end of the interval, all inputs and outputs vary as a constant k of this linear relationship. Therefore:

Case 1	Case 2	
$\phi(\text{success}) = k/6$	$\phi(\text{success}) = t/6 + 1$	(6)
$\phi(\text{troops}) = k/30$	$\phi(\text{troops}) = t/30 + 1/5$	(7)
$\phi(\text{units}) = k/20$	$\phi(\text{units}) = t/20 + 3/10$	(8)
$\phi(\text{equip}) = k/40$	$\phi(\text{equip}) = t/40 + 15/100$	(9)

For case 1, combining equations (1) through (9)

$$\phi(\text{success})\gamma = -\phi(\text{life})\beta + \phi(\text{troops})\mu + \phi(\text{units})\chi - \phi(\text{equip})\epsilon$$

becomes: $$\frac{k}{6}\left(\frac{-t}{24} + \frac{3}{4}\right) = -\phi(\text{life})\left(\frac{-5t}{600} + \frac{1}{5}\right) + \frac{k}{30}\left(\frac{2t}{600} + \frac{9}{10}\right)$$

$$+ \frac{k}{20}\left(\frac{2t}{600} + \frac{9}{10}\right) - \frac{k}{40}\left(\frac{-t}{60} + \frac{4}{10}\right).$$

$$\frac{\phi(\text{life})}{k} = \frac{55t - 432}{60(-t+24)}$$ which has a minimum at t=0

This result is reasonable and shows that the factors of success dominates over the other factors.

For case 2:

$$\phi(\text{life}) = \frac{55t^2 - 282t - 3672}{12(-5t + 120)}$$ which has a minimum at t = 4.19 days.

Paragraph 9.2 through 9.6, which follow are all variations of example 1, with reverse trends of the probabilities of input and output factors for analysis.

9.2 Example 2

TABLE 6

EXAMPLE 2 SUMMARY

INPUTS/OUTPUTS	t=o IMPORTANCE	FUNCTION CHARACTERISTICS	t=6 IMPORTANCE
Success	$z_o = 0.50$	$\gamma = t/24 + 1/2$ (increase)	$z_6 = 0.75$
Life Loss	$y_o = 0.15$	$\beta = 5t/600 + 15/100$ (increase)	$y_6 = 0.20$
Equipment	$l_o = 0.92$	$\chi = -2t/600 + 92/100$ (decrease)	$l_6 = 0.90$
Troops	$k_o = 0.92$	$\psi = -2t/600 + 92/100$ (decrease)	$k_6 = 0.90$
Equipment Loss	$x_o = 0.30$	$\varepsilon = t/60 + 3/10$ (increase)	$x_6 = 0.40$

Note: k_1 k_2 k_3 and $\emptyset$ mean value levels are the same as in Table 5

For this example all trends (slopes) are the negative of those used in example 1. Combining equations (1) through (9) with the parameters of table 6: (case 1)

$$\frac{k}{6}\left(\frac{t}{24} + \frac{1}{2}\right) = -\emptyset(\text{life})\left(\frac{5t}{600} + \frac{15}{100}\right) + \frac{k}{30}\left(\frac{-2t}{600} + \frac{92}{100}\right) + \frac{k}{20}\left(\frac{-2t}{600} + \frac{92}{100}\right) + \frac{-k}{40}\left(\frac{t}{60} + \frac{3}{10}\right).$$

$\dfrac{\emptyset(\text{life})}{k} = \dfrac{-55t - 48}{12(5t+90)}$ which takes on a minimum value at the end-point t=6. The relative importance of success dominates as in example 1.

For case 2:

$$\emptyset(\text{life}) = \frac{-55t^2 - 432t - 612}{12(5t+90)}$$

which has a minimum value for the interval at t=6.

9.3 Example

TABLE 7

EXAMPLE 3 SUMMARY

INPUTS/OUTPUTS	t=o IMPORTANCE	FUNCTION CHARACTERISTICS	t=6 IMPORTANCE
Success	$z_o = 0.75$	$\gamma = -t/24 + 3/4$ (decrease)	$z_6 = 0.50$
Life Loss	$y_o = 0.15$	$\beta = 5t/600 + 15/100$ (increase)	$y_6 = 0.20$
Equipment	$l_o = 0.92$	$\chi = -2t/600 + 92/100$ (decrease)	$l_6 = 0.90$
Troops	$k_o = 0.92$	$\psi = -2t/600 + 92/100$ (decrease)	$k_6 = 0.90$
Equipment Loss	$x_o = 0.30$	$\varepsilon = t/60 + 3/10$ (increase)	$x_6 = 0.40$

Note: k_1, k_2, k_3 and $\emptyset$ mean value levels are the same as in Table 5

For this example all trends except for the success function are the negative of those used in example 1. Combining equations (1) through (9) with the parameters of table 7:

Case 1:

$$\frac{k}{6}\left(\frac{-t}{24} + \frac{3}{4}\right) = -\emptyset(\text{life})\left(\frac{5t}{600} + \frac{15}{100}\right) + \frac{k}{30}\left(\frac{-2t}{600} + \frac{92}{100}\right)$$

$$+ \frac{k}{20}\left(\frac{-2t}{600} + \frac{92}{100}\right) - \frac{k}{40}\left(\frac{t}{60} + \frac{3}{10}\right).$$

$\frac{\emptyset(\text{life})}{k} = \frac{45t - 302}{12(5t+90)}$ which takes on a minimum value at t=0. As before the relative importance of success dominates the other factors.

Case 2:

$\emptyset(\text{life}) = \frac{45t^2 - 132t - 2412}{12(5t+90)}$ which has a minimum at t=0.

9.4 Example 4

TABLE 8

EXAMPLE 4 SUMMARY

INPUTS/OUTPUTS	t=o IMPORTANCE	FUNCTION CHARACTERISTICS	t=6 IMPORTANCE
Success	$z_o = 0.50$	$\gamma = t/24 + 1/2$ (increase)	$z_6 = 0.75$
Life Loss	$y_o = 0.20$	$\beta = -5t/600 + 1/5$ (decrease)	$y_6 = 0.15$
Equipment	$l_o = 0.92$	$\chi = -2t/600 + 92/100$ (decrease)	$l_6 = 0.90$
Troops	$k_o = 0.92$	$\psi = -2t/600 + 92/100$ (decrease)	$k_6 = 0.90$
Equipment Loss	$x_o = 0.30$	$\epsilon = t/60 + 3/10$ (increase)	$x_6 = 0.40$

Note: k_1, k_2, k_3 and $\emptyset$ mean value levels are the same as in Table 5.

For this example all trends except for the life loss function are the negative of those used in example 1. Combining equations (1) through (9) with the parameters of table 8:

Case 1:

$$\frac{k}{6}\left(\frac{t}{24}+\frac{1}{2}\right) = -\emptyset(\text{life})\left(\frac{-5t}{600}+\frac{1}{5}\right)+\frac{k}{30}\left(\frac{-2t}{600}+\frac{92}{100}\right)$$

$$+\frac{k}{20}\left(\frac{-2t}{600}+\frac{92}{100}\right)-\frac{k}{40}\left(\frac{t}{60}+\frac{3}{10}\right).$$

$$\frac{\emptyset(\text{life})}{k} = \frac{-55t-102}{12(-5t+120)}$$ which assumes a minimum value at t=6.

The functions of success and life loss dominate the other functions.

Case 2:

$$\emptyset(\text{life}) = \frac{-55t^2-432t-612}{12(-5t+90)}$$ which has a minimum at t=6.

9.5 Example 5

TABLE 9

EXAMPLE 5 SUMMARY

INPUTS/OUTPUTS	$t=o$ IMPORTANCE	FUNCTION CHARACTERISTICS	$t=6$ IMPORTANCE
Success	$z_o = 0.50$	$\gamma = t/24 + 1/2$ (increase)	$z_6 = 0.75$
Life Loss	$y_o = 0.15$	$\beta = 5t/600 + 15/100$ (increase)	$y_6 = 0.20$
Equipment	$l_o = 0.92$	$\chi = -2t/600 + 92/100$ (decrease)	$l_6 = 0.90$
Troops	$k_o = 0.92$	$\psi = -2t/600 + 92/100$ (decrease)	$k_6 = 0.90$
Equipment Loss	$x_o = 0.40$	$\tau = -t/60 + 4/10$ (decrease)	$x_6 = 0.30$

Note: k_1, k_2, k_3 and $\emptyset$ mean value levels are the same as in Table 5.

For this example all trends except for the equipment loss function are the negative of those used in example 1. Combining equations (1) through (9) with the parameters of table 9:

Case 1:

$$\frac{k}{6}\left(\frac{t}{24} + \frac{1}{2}\right) = -\emptyset(\text{life})\left(\frac{5t}{600} + \frac{15}{100}\right) + \frac{k}{30}\left(\frac{-2t}{600} + \frac{92}{100}\right)$$

$$+ \frac{k}{20}\left(\frac{-2t}{600} + \frac{92}{100}\right) - \frac{k}{40}\left(\frac{-t}{60} + \frac{4}{10}\right).$$

$\frac{\emptyset(\text{life})}{k} = \frac{-49t-102}{12(5t+90)}$ which assumes a minimum at $t=6$. The functions of success and equipment loss dominate the other factors.

Case 2:

$\emptyset(\text{life}) = \frac{-55t^2 - 432t - 612}{12(5t+90)}$ which has a minimum at $t=6$.

9.6 Additional Analysis

Since in the previous examples for Case 1, the success function dominated through out, it is considered essential to investigate the minimizing process with the relative importance of success held constant over the time interval. With $z_o = z_6$ the following results are noted:

Example 1: (Case 1)

$\frac{\phi(\text{life})}{k} = \frac{-5t+132}{12(5t-120)}$ which has a minimum at t=6. The loss of life factor now dominates.

Example 2: (Case 1)

$\frac{\phi(\text{life})}{k} = \frac{-5t-102}{12(5t+90)}$ which has a minimum at t=0. The loss of life factor dominates.

Example 3: (Case 1)

$\frac{\phi(\text{life})}{k} = \frac{-5t-102}{12(5t+90)}$ which has a minimum at t=0. The loss of life factor dominates.

Example 4: (Case 1)

$\frac{\phi(\text{life})}{k} = \frac{5t+102}{12(5t-120)}$ which has a minimum at t=6. The loss of life factor dominates.

Example 5: (Case 1)

$\frac{\phi(\text{life})}{k} = \frac{-5t-102}{12(-5t+120)}$ which has a minimum at t=6. The loss of of equipment factor dominates.

9.7 Conclusions

The minimum values occur where expected, especially when linear functions of inputs and outputs are used. The asymptotic behavior of the linear functions will cause a predominance of a factor upon which the optimization depends. A more detailed sensitivity analysis of the minization process with other functions is a thesis in itself.

10. Conclusions, Extensions, Applications and Acknowledgements

10.1 Conclusions.

10.1.1 The predictable decision maker is one whose mean value levels of the decision parameters asymptotically approach a slope which is a function of time.

10.1.2 Value levels and their relative importance for the decision parameters can be determined as functions of time by experiment and trend analysis.

10.1.3 Optimization of the decision rule for equation, section 2 determines a time of action for the decision process for the individual decision maker, giving realistic results when parameters are linear functions of time.

10.1.4 The methods outlined in this thesis are only as reliable as the trend analysis of the decision maker himself.

10.1.5 This theory of the decision process is not to be substituted for the practices of the decision maker, but only can act as a prediction aid of his decision pattern.

10.2 Extensions

10.2.1 Conduct further analysis over longer time periods of the experimental tests and their associated trend relationships.

10.2.2 Investigate optimization of other than linear monotone increasing functions of mean value levels and their associated probabilities.

10.2.3 Investigate the feasibility of applying computer techniques utilizing this theory to the decision process.

10.2.4 Conduct a detailed analysis of the individual whose experimental data shows no trends, but has random-walk characteristics.

10.3 Applications

10.3.1 Service wide experimental testing of value levels over periods of years will indicate the stability of officers as decision makers prior to selection for flag rank.

10.3.2 Commanders will have a secondary method of decision analysis for comparison with those based upon experience alone.

10.3.3 Analysis of enemy commander's traits over time prior to battle, will afford by these methods, a means of predicting their actions. This knowledge would be highly advantageous to our forces.

10.4 Acknowledgements

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APPENDIX I

OUTLINE OF EXPERIMENT 1

Explanation: Happiness is a term which most people attempt to define, and attain over a period of time. As any trait, happiness has factors which aid in its attainment (inputs) and factors which detract from its attainment (outputs).

Purpose: The purpose of the experiment is to get your opinions concerning the factors which make up happiness. Each of the factors can be subdivided into realms or aspects. These, too, must be evaluated.

Procedure:

1) List factors which aid in the attainment of happiness (inputs).
2) List factors which detract from the attainment of happiness (outputs).
3) Relist separately under the headings of input and output, first the factor you consider most important, the second most important, etc., until original lists are exhausted.
4) For each of these factors assign an ingredient weight from 0 to 10 (not necessarily an integer) as to your assessment of the role the factor has toward attainment (either aid or detract from) of happiness.
5) Subdivide each factor (input or output) into realms (aspects) which you consider specifically make up that factor.
6) For each of these realms assign an ingredient weight from 0 to 10 (not necessarily an integer) as to your assessment of the role the realm has toward attainment of the input or output.
7) For each realm estimate the percentage of conscious thought

emphasized in that area. Total for each input or output must add up to 100%.

8) What percentage of a unit period of time do feel happy rather than unhappy?

9) Repeat above test at discrete equal time intervals (recommend 2 days).

WORKSHEET GUIDE EXAMPLE

HAPPINESS

	INPUTS	OUTPUTS
	A	U
STEPS	B	V
	C	W
1 & 2	D	X
	E	Y
		Z

	INPUTS	WEIGHT (0 - 10)	OUTPUTS	WEIGHT (0 - 10)
	B	9	Z	10
STEPS	C	7	W	6
	E	5	V	4
3 & 4	A	3	U	1.5
	D	2	X	0.5
			Y	0.25

INPUT B

	Realms	Ordered Realms	Weight	%
	B1	B3	9	40
STEPS	B2	B5	5	25
	B3	B1	3	20
5, 6 & 7	B4	B4	2	10
	B5	B2	1	5

Do for all inputs and outputs. Indicate the percentage of the time happy. Examples above outlined no way should imply the number of inputs, outputs or realms to be listed, or their weights. This is to be your honest evaluation of the subject. When test is completed an explanation of the reason for experimentation will be given.

APPENDIX II

OUTLINE FOR EXPERIMENT 2

Explanation: Success at work is a desire of most people. As in any resultant, success at work has factors which aid in its attainment (inputs) and factors which detract from its attainment (outputs).

Purpose: The purpose of the experiment is to get your opinion concerning set factors which make up success at work.

Procedure:

1) Indicate on worksheet the starting time and date of testing.
2) The worksheet has listed input/output factors with their respective realms. Relist under input, output (each respectively), the factor you consider most important, second most import, etc., until original lists are exhausted.
3) For each of these factors assign an ingredient weight from 0 to 10 (not necessarily an integer) as to your assessment of the role the factor has toward attainment of success at work.
4) Similarily order each realm and assign an ingredient weight.
5) For each realm estimate the percentage of conscious thought emphasized in that area. Total for each input or output must add up to 100%.
6) Indicate what percentage of time you feel success at work is attained.
7) Repeat: (all times from completion of previous test)

a) 1st time - 15 min.	e) 5th time - 15 min.
b) 2nd time - 30 min.	f) 6th time - 30 min.
c) 3rd time - 1 hour.	g) 7th time - 1 hour.
d) 4th time - 1 day.	h) 8th time - 4 days.

APPENDIX III

Actual Worksheet Experiment 2 for t = 1674 mins.

SUCCESS AT WORK   DATE: 3-26-64   TIME STARTED: 5:15 pm

INPUTS		OUTPUTS	
A.	Capability	U.	Laxity
B.	Education	V.	Ignorance
C.	Attitude	W.	Incapability
D.	Returns	X.	Non-return
E.	Environment	Y.	Unenvironmental

INPUTS	WEIGHTS	OUTPUTS	WEIGHTS
A	8	W	8
B	7	V	7
C	6	U	6
D	5	X	5
E	2.5	Y	2.5

	REALMS	ORDERED REALMS	WEIGHTS	%
	A1 Skill	A3	8	50
INPUT A	A2 Aptitude	A2	7	30
	A3 Intelligence	A1	6	20
	B1 Training	B1	7	50
INPUT B	B2 Cultural Interest	B3	6	30
	B3 Intellectual Enlightment	B2	5	20
	C1 Interest	C4	6	30
	C2 Challenge	C1	5	25
INPUT C	C3 Enthusiasm	C2	4	17.5
	C4 Goals	C3	4	17.5
	C5 Desires	C5	2	10
	D1 Money	D4	5	40
INPUT D	D2 Satisfaction	D1	4	30
	D3 Praise	D2	3	20
	D4 Motivation	D3	1	10
	E1 Employee Relations	E1	4	35
	E2 Company Policies	E3	4	35
INPUT E	E3 Supervision Relations	E2	2	20
	E4 Physical Conditions	E4	1	10

	REALMS	ORDERED REALMS	WEIGHTS	%
	U1 No Interest	U4	6	30
	U2 No Challenge	U1	5	25
OUTPUT U	U3 No Enthusiasm	U2	4	20
	U4 No Goal	U3	4	20
	U5 No Desires	U5	1	5
	V1 Poor Training	V1	7	40
OUTPUT V	V2 Lack of Cultural Interest	V3	6	35
	V3 No Intellectual Enlightenment	V2	5	25
	W1 Lack of Skill	W3	8	40
OUTPUT W	W2 Lack of Aptitude	W2	7	35
	W3 Lack of Intelligence	W1	6	25
	X1 Poorly Paid	X3	6	40
	X2 Little Satisfaction	X1	5	35
OUTPUT X	X3 Poor Motivation	X2	4	20
	X4 No Praise	X4	1	5
	Y1 Poor Employee Relations	Y1	4	35
	Y2 Poor Company Policies	Y3	4	35
OUTPUT Y	Y3 Poor Supervisory Relations	Y2	2	20
	Y4 Poor Physical Conditions	Y4	1	10

% Successful 80%

Time Completed: 5:25

APPENDIX IV

SAMPLE CALCULATIONS

All calculations found in this appendix are based upon the data listed in Appendix III.

Mean Value Level (ϕ_t; t = 6)

Recall definition 1: $\phi_t = \dfrac{\sum_{i=1}^{n} v_i}{n}$

For Input A: $\phi_6(A)=7.0$

For Input B: $\phi_6(B)=6.0$

Adjusted Mean Value Level (ϕ'_t ; t = 6)

Recall definition 2: $\phi'_t = \phi_t$ (weight of input).

For Input A: $\phi'_6(A) = 8(7.0) = 56.0$

For Input B: $\phi'_6(B) = 7(6.0) = 42.0$

Weighted Mean Value Level ($P\phi'_t$; t = 6)

Recall relationship 4, section 5:

$P\phi' = \sum$(realm weight)(% realm concentration).

$P\phi'_6(A) = 8(.5) + 7(.3) + 6(.2) = 7.3$

Relative Importance (P_t ; t = 6)

Recall relationship 3, section 5:

$P_t = \dfrac{\sum\text{(realm weight)(\% realm concentration)}}{\phi'}$.

$P_6(A) = 7.3/56.0 = 0.130$

In general there is no upper limit on the value of the relative importance which may be greater than one, as it is not restricted to an upper limit as is a probability function.

Resultant Value (ϕ (result))

From equation 1, section 2 and as outlined in section 5:

$$\phi\ (\text{result}) = \frac{\sum P\phi'}{P(\text{result})},$$

where P(result) = % successful = 80%

thus, ϕ(result) = -1.06 relative units.